talk by Scott Chaos

CONTACT MFLD WITH FLEXIBLE FILLINGS

—Lazarev.

(M^{2n-1}, ξ) contact mfld. W^{2n} Weinstein filling

$\Rightarrow$ handles have index $\leq n$

THM [Yau] If W_1, W_2 are two subcritical fillings of M ..

$\Rightarrow \quad H^*(W_1) \simeq H^*(W_2)$

THM1 [Lazarev] If W_1, W_2 are two flexible fillings of (Y^{2n-1}, ξ)

$\Rightarrow \quad H^*(W_1) \simeq H^*(W_2)$.

RMK replace W_2 by Liouville filling w/ $SH = 0$.

THM 2: $n \geq 3$ & (Y^{2n-1}, ξ) has flexible filling $\Rightarrow \exists$ ∞-ly (ξ_n) s.t. $\xi_n \underset{\text{isomorphic}}{\not\simeq} \xi$ such that (Y^{2n-1}, ξ_n) has flexible fillings.

Once we have fillings of exotic sphere $(S^{2n-1}, \xi) \underset{\text{ff}}{\subseteq} M$

Take $(Y, \xi) \underset{\text{ff}}{\subseteq} W$ and connect sum. (attaching a 1-handle)

$\uparrow$ flexible fillings

$\Rightarrow$

(Y, ξ_m)

$\cap$

$W \natural M$ (still flexible filling)

Look at $H^*(W \natural M) = H^*(W) \oplus \boxed{H^*(M)}$

For n odd, you can take M_i = filling of Brieskorn mflds

$$\dim(H^{n_i}(M_i)) \to \infty$$

} make them flexible

n even. [Geiges, application of contact surgery]

Start w/ $\dim H^n(M) \geq 1$

$$M_i = \#_i M.$$

REMARK: $(Y, \xi) \overset{hmtp}{\sim} (Y, \xi_m)$.

Proving thm 1:

DEFN: (Y, ξ) is dynamically convex (DC) if $\exists\ \alpha$ such that all Reeb orbits have positive degree

$$|\gamma| = \mu_{CZ}(\gamma) + n - 3 > 0$$

THM If DC $\Rightarrow$ SH^+ independent of Stein filling, (if $c_1(\omega) = 0$)

DEFN: (Y, ξ) is asymptotically dynamically convex (ADC)
If $\exists\ \alpha_1 \geq \alpha_2 \geq \alpha_3 \geq \ldots\ldots$

$$D_1 \leq D_2 \leq D_3 \leq \ldots \to \infty \quad \text{s.t.} \quad p < D_i$$

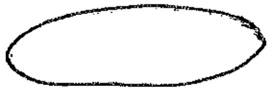
(Y, α_i). where $A(\gamma) = \int_\gamma \alpha < D_i$

$\alpha_1 = f\alpha_2$ etc.

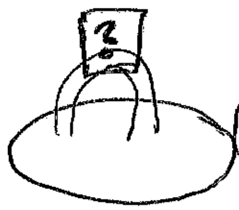
THM (L) (Y, ξ) If ADC $\Rightarrow$ SH^+ independent of Stein filling (if $c_1(\omega) = 0$)

$$\begin{tikzcd} H^* \arrow[rr] & & SH^* = 0 \arrow[dl] \\ & (SH^+)^* \arrow[ul, "\sim"] & \end{tikzcd}$$

If (Y^{2n-1}, ξ) has a flexible filling W



(Y^{2n-1}, ξ')



(S^{2n-1}, ξ_{std})

THM If $(Y_-, \xi) \to (Y_+, \xi)$ then ADC $\Rightarrow$ ADC

subcritical (Yau)

flexible (Oleg)

this doesn't work necessarily if we change ADC to DC here.

PROPOSITION (BEE) after a surgery along an attaching legendrian sphere Λ^{n-1} $(n \geq 3)$

then {new Reeb orbits w/ period < 1 (action)

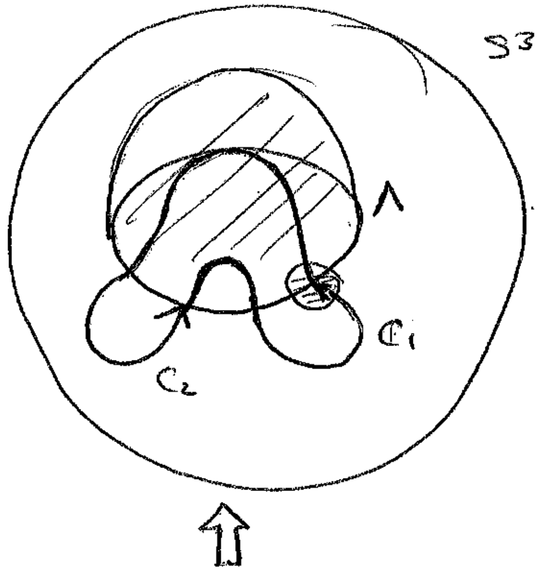
$\updownarrow$

{cyclic words of Reeb chords in Y}. w/ action $< D$

[Yau] $\|\gamma_{c_1 \cdots c_n}\| = \sum_i \|c_i\| + n - 3$

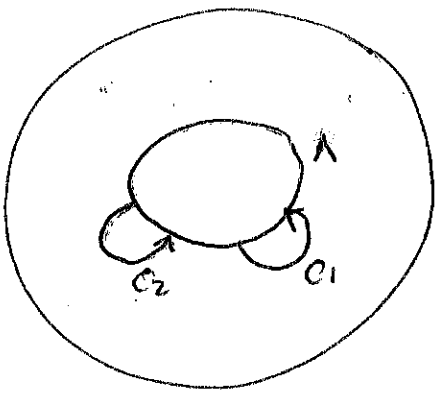
$\Lambda = S^1 \subseteq S^3$

Reeb chord = start at a point on legendrian and trace out path along Reeb v.f.



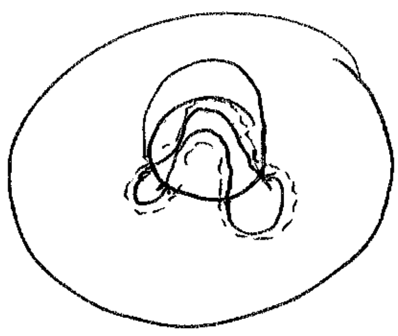
we are attaching a 2-handle to Λ

orbit doesn't even have to touch legendrian.



every orbit leaves the 2 handle b/c there are no closed geodesics on the disk.

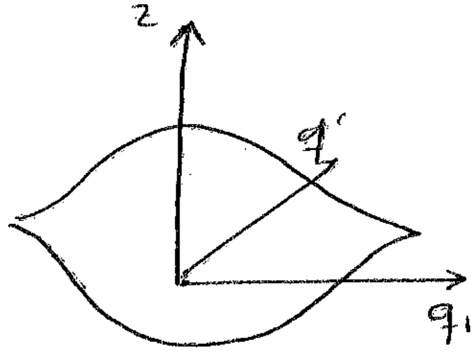
Now what if you perturb chords, the closing up of the new reeb chords should be close to the 1st one



Key Lemma: If $\Lambda^{n-1} \leq Y$ $(n \geq 3)$ be a loose legendrian
$\Rightarrow \exists$ ~~[illegible]~~ legendrian isotopy s.t. Reeb chords have (periodic band) + deg.

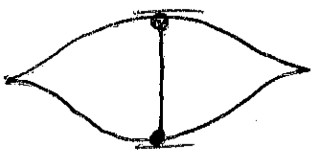
Use a local model on $(\mathbb{R}^{2n-1}, dz + p_1 dq_1 + p_2 dq_2 + \dots)$
front projection

$(z, q_1, q_2, \dots)$



$p_i = \frac{dz}{dq_i}$

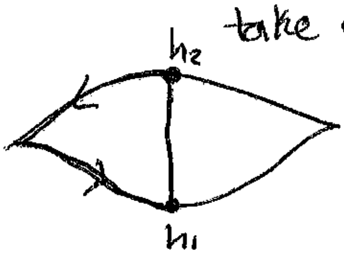
What is a Reeb chord on a front projection



we need the slopes to be the same.

What is the degree of a Reeb chord?

take a path btwn h_2, h_1

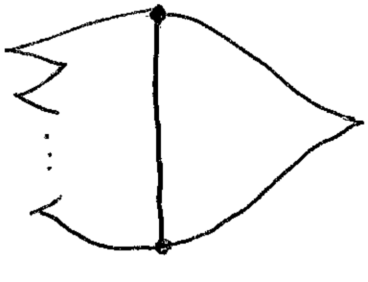


deg = # Down cusps − # up cusps + $\text{ind}_{h_2 - h_1}(p) - 1$

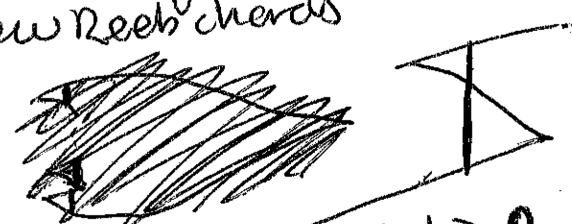
here deg = 1 − 0 + 2 − 1 = 2

defined up to maslov of the knot b/c it depends on choice of path

Now, lets increase degree by ~~a~~ stabilizing.



stabilizing also adds in new Reeb chords



ind ≥ 0

Since Λ is loose you can realize stabilizations by isotopy.