SARA : APPLICATIONS OF WENDL'S THM TO CLASSIFICATIONS OF FILLINGS

Recall:

$\{$Stein fillings$\}$ $\longleftrightarrow$ $\{$positive factorizations of monodromies of compatible open books$\}$

Not easy to consider all compatible open books. But we can just look at planar OBD:

THM: If (Y,ξ) admits a planar OBD $\Rightarrow$ every strong symplectic filling is symplectic deformation equivalent to a blowup of an allowable LF, compatible w/ open book.

DEFINITION: $L(p,1)$ is a quotient of S^3 by $\mathbb{Z}_p$ action

$$(z_1, z_2) \longmapsto (e^{2\pi i/p} z_1,\ e^{2\pi i/p} z_2)$$

DEFINITION: A contact structure ξ is overtwisted if it contains an overtwisted disk. Otherwise ξ is tight.

If (Y,ξ) is a tight contact structure & if $(\tilde{Y},\tilde{\xi})$ is tight then we call ξ universally tight. Otherwise ξ is virtually OT. (VOT)

[This talk is based on: Plamenevskaya / Van Horn Morris. Planar open books, monodromy factorizations & symplectic Fillings.]

THM Every VOT contact structure on $L(p,1)$ has a unique Stein filling up to symp def which is also its unique weak filling up to symplectic def & blow up.

COROLLARY:
(If $p \neq 4$ then we can replace VOT with tight)

(Honda 2000) The tight contact structures on $L(p,1)$ are $\xi_1, \ldots, \xi_{p-1}$ where ξ_k arises from legendrian surgery on an unknot, with stabilization $p-2$ times, and k cusps on the left.

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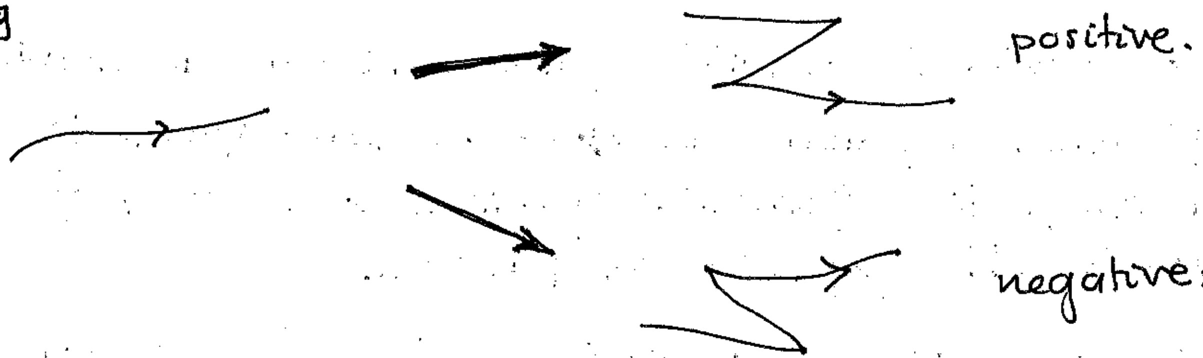
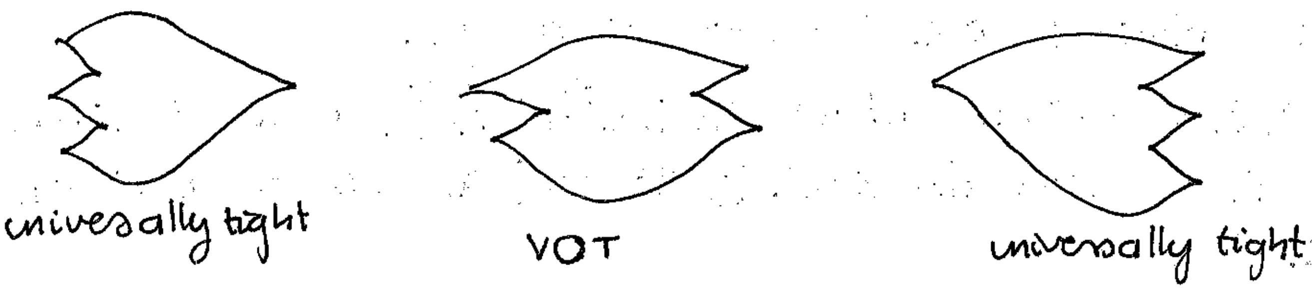


- $\xi_2, \ldots, \xi_{p-2}$ VOT.

 STABILIZED 2 TIMES $(p=4)$



In open book from legendrian surgery for ξ_k

page = disk w/ $(p-1)$ holes $\quad$ let $n = (p-1)$

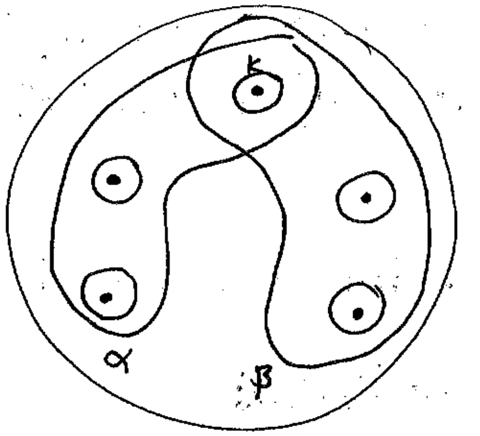
δ_j = curve around jth hole

$$\Phi = D_\alpha D_{\delta_1} \cdots D_{\delta_{k-1}} D_{\delta_{k+1}} \cdots D_{\delta_n} D_\beta$$

α contains holes $1, \ldots, k$

β contains holes $k, \ldots, n$

For instance,



holes $1, \dots, k \to$ one type of stab
$k, \dots, n \to$ the other type of stab.

$\Rightarrow$ Stein filling: $D^4 \cup$ 2-handle.

LEMMA 1: Any positive factorization of Φ takes the form

$$D_{\alpha'} D_{\delta_1} \cdots D_{\delta_{k-1}} D_{\delta_{k+1}} \cdots D_{\delta_n} D_{\beta'}$$

where α', β' enclose the same holes as α, β. $\boxed{(k \geq 1)?}$

(Margalit / McCammond 2009) If $\mathbb{D}_n$ is the disk w/ n holes

$\Rightarrow$ $MCG(\mathbb{D}_n)$ has a presentation with generators all convex
mapping class group

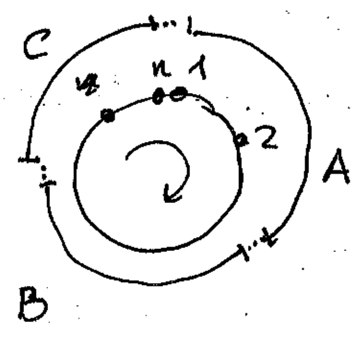
Dehn twists & relations:

Dehn twists around disjoint curves commute.

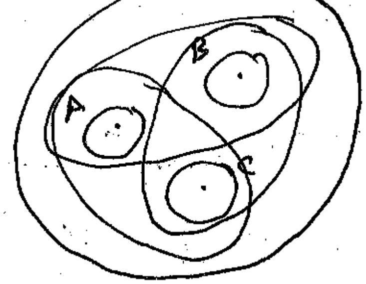
(i) Dehn twists around disjoint curves commute.

(ii) lantern relations

$$D_A D_B D_C D_{A \cup B \cup C} = D_{A \cup B} D_{A \cup C} D_{B \cup C}$$



$A = \{a_1, \dots, a_j\}$



γ simple closed curve is convex if it is ~~cvx~~ isotopic to ∂ of convex hull of a set of holes

convex Dehn twist = Dehn twist around convex γ.

Proof of lemma 1: ~~[illegible]~~

If γ is convex, factor Dehn twist around γ D_γ into a product of boundary & pairwise Dehn twist (not nec positive)

$\underset{\text{go around one hole}}{\text{boundary}}$ $\underset{\text{go around a pair of holes}}{\text{pairwise}}$

can do this by lantern relations, etc.

Multiplicity of boundary or pairwise Dehn twist invariant under lantern relation & its in the form we want.

can do this also for a generic curve γ. So we can factor any element in the mapping class group.

LEMMA 2 If $\Psi = D_{\alpha'} D_{\delta_1} \cdots D_{\delta_{k-1}} D_{\delta_{k+1}} \cdots D_{\delta_n}$

$\Rightarrow$ OB w/ pages = $\mathbb{D}_n$ & this monodromy ~~exactly~~ represents (S^3, ξ_{std})

The knot in S^3 induced by β' is an unknot with framing $(-p+1)$

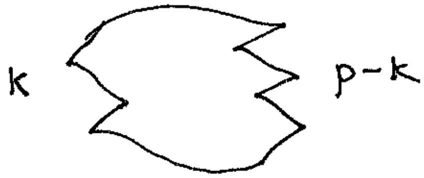
Proof in paper

Proof of Thm Let X be a LF for an arbitrary factorization of Φ

$$D_{\alpha'} D_{\delta_1} \cdots D_{\delta_k} D_{k+1} \cdots D_{\delta_n} D_{\beta'}$$

by lemma 2, X diffeo to B^4 w/ 2 handle attached along the unknot w/ framing $-(p-1)$. So X has Stein

structure from surgery on a knot with $tb = -p+1$
up to Legendrian isotopy is the only way to get
$(L(p,1), \xi_k)$.

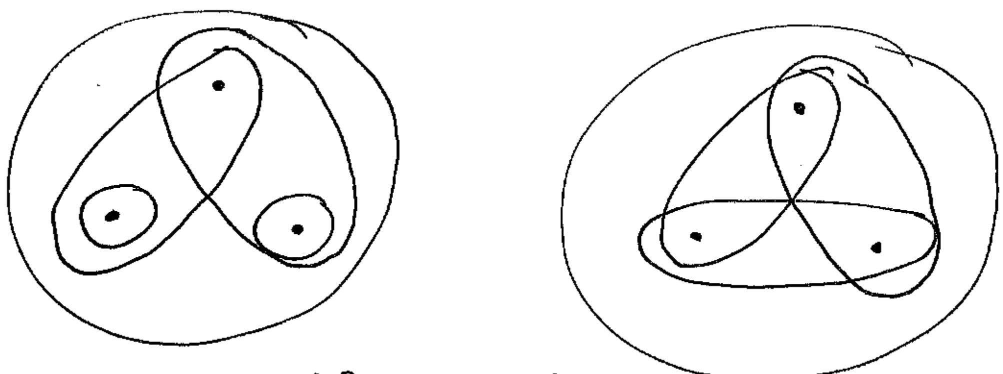


compatible symplectic structure is unique up to symp def
(Gompf 2004)

$\Rightarrow$ All stein structures on X are the same. up to symp def.

$\Rightarrow$ Stein fillings are unique. □
(Wendl's thm)

CAREFUL
$p = 4 \Rightarrow n = p-1 = 3$



these give different stein fillings.

only for $p = 4$.