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Agustin - Strongly fillable contact manifolds and J-holomorphic foliations

A contact manifold (M,ξ) is planar if it admits an OBD supporting ξ with genus 0 pages. A LF is allowable if all vanishing cycles are homologically non trivial. If $\pi : M\setminus B \to S^1$ is an OBD, denote $\hat{\pi} : M \setminus N(B) \to S^1$, where $N(B)$ = nbhd of B.

Definition: for $\pi : M\setminus B \to S^1$ OBD, a contact form α is Giroux if $d\alpha|_{page} > 0$ and α is positive on B. We say that $\xi = \ker\alpha$ is supported by the OBD.

Rem: it is the case if ξ is almost tangent to the pages.

Reference: Wendl's paper "[title of talk]"

Theorem 1: Suppose (W,ω) is a strong symplectic filling of a planar contact manifold (M,ξ), and $\pi : M\setminus B \to S^1$ is a planar OBD supporting ξ. Then, $\exists$ some enlarged version of W, called (W',ω), obtained by a trivial symplectic cobordism to W, such that W' admits a symplectic LF $\Pi : W' \to \mathbb{D}$ such that $\Pi|_{\partial W' \setminus N(B)} = \hat{\pi}$. Moreover, $\Pi : W' \to \mathbb{D}$ is allowable if W is minimal.

Corollary 1: every strongly fillable planar contact manifold is also Stein fillable.

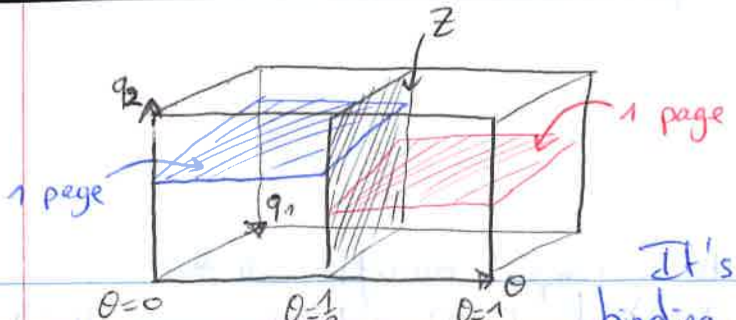
So: strongly but not Stein fillable $\Rightarrow$ not planar.

Theorem 2: Suppose (W,ω) strong symplectic filling of (T^3, ξ_0), where $\xi_0 = \ker(\cos(2\pi\theta)dq_1 + \sin(2\pi\theta)dq_2)$. Then, one can attach to W a trivial symplectic cobordism such that the enlarged filling W' admits a symplectic LF $\Pi : W' \to [0,1]\times S^1$ for which $\Pi|_{\partial W' \setminus N(Z)} = \hat{\pi}_0$. Moreover, every singular fiber is the union of an annulus with an exceptional sphere; in particular, if (W,ω) is minimal, there are no singular fibers.

$$T^3 = S^1\times S^1 \times S^1 = T^2 \times S^1 \ni (q_1, q_2, \theta)$$

$Z = \{\theta \in \{0, 1/2\}\} \subset T^3$, and π_0 is

$$\pi_0 : T^3\setminus Z \to \{0,1\}\times S^1 : (q_1,q_2,\theta) \mapsto \begin{cases} (0,q_2) & \text{if } \theta\in(0,1/2) \\ (1,q_2) & \text{if } \theta \in (1/2,1) \end{cases}$$



It's kind of like an OB, but the binding is a T^2, that we should see as a "blow up" of S^1.

Wendl's paper numbering

Theorem 4: all minimal strong fillings of T^3 are symplectically deformation equivalent, and every exact filling of T^3 is symplectomorphic to a star-shaped domain in (T^*T^2, ω_0), ie something of the form $\{(q, tf(q,p)p) \in T^*T^2 \mid t \in [0,1), (q,p) \in S^*T^2, f: S^*T^2 \to (0,\infty)\}$.

Corollary: every minimal strong filling of T^3 (in particular, Stein fillings) is diffeomorphic to $T^2 \times D^2$.

Theorem 5: the group $\mathrm{Symp}_c(T^*T^2, \omega_0)$ of compactly supported symplectomorphisms of (T^*T^2, ω_0) is contractible.

Now, an important compactness theorem, for all these results:

Definitions: * a finite energy foliation $\mathcal{F}$ on $(M_0, \lambda$ contact form, J a $\mathbb{C}$-str$)$ is a foliation of $\mathbb{R} \times M_0$, such that

1) $\forall u \in \mathcal{F}$ leaf, any $\mathbb{R}$-translation of u is also a leaf.
2) every $u \in \mathcal{F}$ is the image ~~is the image~~ of a finite energy J-hol. curve satisfying some uniform energy bound.

* A leaf $u \in \mathcal{F}$ is interior if it is not a trivial cylinder and all its ends belong to Morse-Bott submanifolds in M_0.
* $\mathcal{F}$ is positive if every leaf $\neq$ trivial cylinder has only positive ends.
* $u \in \mathcal{F}$ is stable if it has genus 0, all punctures are odd, and its index is 2. (1-param for $\mathbb{R}$ translation, 1 for family on the base)
* $u \in \mathcal{F}$ is asymptotically simple if all of its asymptotics are simply covered, and belong to pairwise disjoint Morse-Bott submanifolds.

Holomorphic curves & compactness.

(M^3, ξ) with Morse-Bott λ, J_+ ~~a~~ λ-compatible

$M_0^\varepsilon \subseteq M$ compact submanifold with Morse-Bott boundary

$\mathcal{F}_+$ positive finite energy foliation on (M_0, λ, J_+) containing an interior stable leaf that is asymptotically simple

Also, let (W^∞, ω) be a compact manifold such that
$W^\infty = W \cup_{\partial W} ([R,\infty) \times M)$ for $R \in \mathbb{R}$ and $\omega|_{[R,\infty)\times M} = d(e^a \lambda)$.
$\overline{W}^\infty = W \cup_{\partial W} ([R,\infty] \times M)$, $\partial \overline{W}^\infty = M$.
Choose J on W^∞ ω-compatible st $J|_{[a_0,\infty)\times M} = J_+$ for $a_0 \in [R,\infty)$.
Let $\mathcal{F}_0 = \bigcup$ leaves in $\mathcal{F}_+$ lying in $[a_0,\infty)\times M$.

Let $\mathcal{M}$ = moduli space of finite energy J-curves in W^∞, and let $\overline{\mathcal{M}}$ be a compactification of $\mathcal{M}$. Let $u_0 \in \mathcal{F}_0$ an asymptotically simple curve, and let $\mathcal{M}_0 \subseteq \mathcal{M}$ connected compact containing u_0.